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The Penrose limit of AdS×S space and holography[1]

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Abstract

In the Penrose limit, AdS×S space turns into a Cahen-Wallach (CW) space whose Killing vectors satisfy a Heisenberg algebra. I discuss how this algebra is mapped onto the holographic screen on the boundary of AdS. Furthermore, I show that the algebra on the boundary of AdS may be obtained directly from the CW space by appropriately constraining the states defined on it. By viewing the constraints as generators of gauge transformations, I obtain a "holographic screen" on the CW space as a gauge-fixing condition.

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The Penrose limit [1, 2] of AdS$\times S$ space is obtained by boosting along a null geodesic. If the spin in S is non-vanishing, then one obtains a Cahen-Wallach (CW) space whose Killing vectors satisfy a Heisenberg algebra. The sigma model (string theory) defined on such a background is exactly solvable [3–22] and may therefore shed some light on the AdS/CFT correspondence. Naturally, it attacted much attention resulting in an extensive literature on the subject which cannot be adequately cited here.

The existence of an AdS/CFT correspondence in the Penrose limit has presented a puzzle because there is no apparent holographic screen in the CW space on which the CFT would reside, unlike its AdS counterpart [23]. A number of proposals on this issue have been made [24–27]. In the case of pure AdS space, the Penrose limit is flat Minkowski space. This is a special case of AdS$\times S$ in which we consider a geodesic with no spin in S. Even though no holographic principle exists on a Minkowski space, a "holographic screen" can be obtained upon restricting the states on the Minkowski space to those with a certain fixed scaling dimension. By viewing this as a constraint generating gauge transformations, we may introduce a hypersurface as a gauge-fixing condition. The Poison brackets ought to be replaced by Dirac brackets. This turns the Poincaré algebra on the flat Minkowski space into a conformal algebra in the limit in which the hypersurface becomes flat. The hypersurface thus defined plays the role of the holographic screen [28].

Here, I extend the discussion in [28] to generic null geodesics in AdS$\times S$ space. First, I discuss how the isometries of CW space which form a Heisenberg algebra are mapped onto the holographic screen where they act on states of large scaling dimension Δ and R-charge J with small (fixed) $\Delta - J$. I then show that the holographic screen may be obtained by an appropriate restriction of the Hilbert space on CW space. This restriction involves scale transformations (states are required to have a fixed scaling dimension) which may be viewed as gauge transformations. Then the gauge-fixing condition is a restriction to a hypersurface in CW space. In the limit where this hypersurface becomes flat, I recover the expected state of affairs on the AdS holographic screen.

Let me start by fixing the notation. I am interested in the AdS$_{p+1} \times S^q$ space. AdS$_{p+1}$ is defined within a flat $(p+2)$-dimensional space as the hypersurface

$$X_0^2 - X_1^2 - \ldots - X_p^2 + X_{p+1}^2 = R^2 \tag{1}$$

S^q is the surface

$$Y_1^2 + \ldots + Y_{q+1}^2 = R^2 \tag{2}$$

in the $(q+1)$-dimensional Euclidean space $\mathbb{E}^{q+1}$. The respective metrics in the two embedding spaces are

$$ds^2_{\mathrm{AdS}} = -dX_0^2 + dX_1^2 + \ldots + dX_p^2 - dX_{p+1}^2 \quad , \quad ds^2_S = dY_1^2 + \ldots + dY_{q+1}^2 \tag{3}$$

Expressing the embedding coordinates in terms of global parameters spanning AdS_{p+1},

$$X^0 = R\cosh\rho\cos t \quad , \quad X^{p+1} = R\cosh\rho\sin t \quad , \quad X^i = R\sinh\rho\Omega^i \quad (i = 1, \ldots, p) \tag{4}$$

where $\Omega_1^2 + \cdots + \Omega_p^2 = 1$ span the unit sphere S^{p-1}, the AdS_{p+1} metric, which is inherited from the embedding upon imposing the constraint (1), may be written as

$$\frac{ds^2_{\mathrm{AdS}}}{R^2} = -\cosh^2\rho\, dt^2 + d\rho^2 + \sinh^2\rho\, d\Omega^2_{p-1} \tag{5}$$

Similarly, we parametrize the sphere S^q by

$$Y^1 = R\cos\theta\cos\phi \quad , \quad Y^q = R\cos\theta\sin\phi \quad , \quad Y^i = R\sin\theta\tilde{\Omega}^i \quad (i = 2, \ldots, q-1) \tag{6}$$

where $\tilde{\Omega}_2^2 + \cdots + \tilde{\Omega}_{q-1}^2 = 1$ span the unit sphere S^{q-2}. The metric on the sphere reads

$$\frac{ds^2_S}{R^2} = \cos^2\theta\, d\phi^2 + d\theta^2 + \sin^2\theta\, d\tilde{\Omega}^2_{q-2} \tag{7}$$

The group of isometries of AdS_{p+1} is $SO(p,2)$ whose generators we shall denote by K_{AB}. In terms of the global AdS coordinates (4), we have

$$\begin{aligned} K_{(p+1)0} &= i\partial_t \\ K_{0i} &= \cos t\,(\Omega_i\partial_\rho + \coth\rho\nabla_i) - \sin t\tanh\rho\Omega_i\partial_t \\ K_{(p+1)i} &= \sin t\,(\Omega_i\partial_\rho + \coth\rho\nabla_i) + \cos t\tanh\rho\Omega_i\partial_t \end{aligned} \tag{8}$$

the rest of the generators K_{ij} $(i,j=1,\ldots,p)$ being the angular momenta on the sphere S^{p-1} spanned by Ω^i. The algebra is easily verified to be $SO(p,2)$ if one uses

$$\nabla_i \Omega_j = \delta_{ij} - \Omega_i \Omega_j \tag{9}$$

It is also useful to introduce the ladder operators

$$K_{\pm i} = K_{0i} \pm i K_{(p+1)i} = e^{\pm it} \left(\Omega_i (\partial_\rho \pm i \tanh \rho \partial_t) - \coth \rho \nabla_i \right) \tag{10}$$

in terms of which the $SO(p,2)$ algebra reads

$$[\, K_{0(p+1)} \,,\; K_{\pm i} \,] = \pm K_{\pm i} \quad , \quad [\, K_{+i} \,,\; K_{-j} \,] = 2i K_{ij} + 2\delta_{ij} K_{0(p+1)} \tag{11}$$

The quadratic Casimir is

$$\begin{aligned} \mathcal{C}_{\text{AdS}} &= \tfrac{1}{2} K^{AB} K_{AB} \\ &= K_{0(p+1)}^2 - \tfrac{1}{2} \{\, K_{+i} \,,\, K_-^{\ i} \,\} + \tfrac{1}{2} K_{ij} K^{ij} \\ &= -\partial_\rho^2 - \frac{p \cosh^2 \rho - 1}{\cosh \rho \sinh \rho}\, \partial_\rho + \frac{1}{\cosh^2 \rho}\, \partial_t^2 - \frac{1}{\sinh^2 \rho}\, \nabla_\Omega^2 \end{aligned} \tag{12}$$

Setting $\mathcal{C}_{\text{AdS}} = -m^2 R^2$, we deduce the scalar wave equation on AdS_{p+1},

$$\left(\partial_\rho^2 + \frac{p \cosh^2 \rho - 1}{\cosh \rho \sinh \rho}\, \partial_\rho - \frac{1}{\cosh^2 \rho}\, \partial_t^2 + \frac{1}{\sinh^2 \rho}\, \nabla_\Omega^2 \right) \Psi^{(\text{AdS})} = m^2 R^2\, \Psi^{(\text{AdS})} \tag{13}$$

Near the boundary of AdS ($\rho \to \infty$), we have $\mathcal{C}_{\text{AdS}} \approx \partial_\rho^2 + p\partial_\rho$. If $\Psi \sim \rho^{-\Delta}$ near the boundary, we deduce

$$\mathcal{C}_{\text{AdS}} = \Delta(\Delta - p) = m^2 R^2 \tag{14}$$

and so

$$\Delta = \Delta_\pm = \tfrac{1}{2} p \pm \tfrac{1}{2} \sqrt{p^2 + 4m^2 R^2} \tag{15}$$

The normalizable modes have $\Delta = \Delta_+$. The exact solution to the wave equation is

$$\Psi_{nL\vec{m}}^{(\text{AdS})} = e^{i\omega t}\, \tanh^L \rho\, \cosh^{-\Delta} \rho\; {}_2F_1(-n, L+\Delta+n; L+p/2; \tanh \rho) Y_{L\vec{m}}(\Omega_{p-1}) \tag{16}$$

where $\omega = \Delta + L + 2n$ $(n = 0,1,2,\ldots)$, labeled by the quantum number n (or equivalently, ω) and the $SO(p)$ quantum numbers $(L, \vec{m})$. The group of isometries $SO(p,2)$ has a maximum compact subgroup $SO(2) \times SO(p)$, where $SO(2)$ is generated by H, defined as the momentum conjugate to t ($H = i\partial_t$), and $SO(p)$ is the group of rotations. The solutions to the wave equation form a highest-weight representation of $SO(p,2)$, the highest-weight state being the one with $n=0$. It has energy Δ, where $H = -i\partial_t$, and transforms trivially under the group of rotations $SO(p)$, i.e., it has $L=0$. Explicitly,

$$\Psi_0^{(\mathrm{AdS})} = e^{i\Delta t}\ \cosh^{-\Delta}\rho \tag{17}$$

The solution space is built by repeatedly acting on the ground state with the creation operators K_{+i}. The boundary is conformally equivalent to an Einstein universe $S^1\times S^{p-1}$ and the $SO(p,2)$ generators turn into the generators of the conformal group,

$$\mathcal{K}_{(p+1)0} = i\partial_t \quad , \quad \mathcal{K}_{\pm i} = e^{\pm it}\left(-\Delta\Omega_i + \nabla_i \pm i\Omega_i\partial_t\right) \tag{18}$$

together with the $SO(p)$ generators $\mathcal{K}_{ij} = K_{ij}$.

The generators of $SO(q)$, which is the group of isometries of S^q, are J_{MN}, where

$$\begin{aligned} iJ_{1q} &= \partial_\phi \\ iJ_{1i} &= \cos\phi\left(\tilde\Omega_i\partial_\theta + \cot\theta\nabla_i\right) + \sin\phi\tan\theta\tilde\Omega_i\partial_\phi \\ iJ_{qi} &= \sin\phi\left(\tilde\Omega_i\partial_\theta + \cot\theta\nabla_i\right) - \cos\phi\tan\theta\tilde\Omega_i\partial_\phi \end{aligned} \tag{19}$$

($\tilde\Omega_2^2+\ldots+\tilde\Omega_{q-1}^2 = 1$) the rest of the generators J_{ij} ($i,j = 2,\ldots,q-1$) being the angular momenta on the sphere S^{q-2} spanned by $\tilde\Omega^i$. The scalar wave equation is

$$\frac{1}{\cos\theta\sin^{q-2}\theta}\,\frac{\partial}{\partial\theta}\left(\cos\theta\sin^{q-2}\frac{\partial\Psi^{(S)}}{\partial\theta}\right) + \frac{1}{\cos^2\theta}\,\partial_\phi^2\,\Psi^{(S)} + \frac{1}{\sin^2\theta}\,\nabla^2_{\tilde\Omega_{q-2}}\Psi^{(S)} = -\mathcal{C}_S\,\Psi^{(S)} \tag{20}$$

where $\mathcal{C}_S = J(J+q)$ is the quadratic Casimir for the sphere S^q,

$$\mathcal{C}_S = \tfrac12 J^{MN}J_{MN} \tag{21}$$

The heighest-weight state is the eigenfunction of the "Hamiltonian" $-i\partial_\phi$ corresponding to the lowest eigenvalue $-J$,

$$\Psi_0^{(S)} = e^{-iJ\phi}\cos^J\theta \tag{22}$$

The other states are obtained by acting with the ladder operators

$$J_{+i} = J_{1i} + iJ_{qi} = i\,e^{i\phi}\left(\tilde\Omega_i(\partial_\theta - i\tan\theta\partial_\phi) + \cot\theta\nabla_i\right) \tag{23}$$

The Penrose limit can be obtained as the scaling limit $R \to \infty$ and $\rho = r/R$, $t = t_- - t_+/R^2$, $\theta = u/R^2$, $\phi = t_- + t_+/R^2$. The metric in AdS space (5) turns into

$$ds^2_{AdS} = -(R^2 + r^2)dt^2 + dr^2 + r^2 d\Omega^2_{p-1} + o(1/R^2) \tag{24}$$

and the sphere (7) goes to

$$ds^2_S = (R^2 - u^2)d\phi^2 + du^2 + u^2 d\tilde{\Omega}^2_{q-2} + o(1/R^2) \tag{25}$$

The combined space is Cahen-Wallach space in Brinkman form,

$$ds^2_{AdS} + ds^2_S \to ds^2_{CW} = 4dt^+ dt^- - (r^2 + u^2)(dt^-)^2 + dr^2 + r^2 d\Omega^2_{p-1} + du^2 + u^2 d\tilde{\Omega}^2_{q-2} \tag{26}$$

The Killing vectors turn into

$$\begin{array}{rcccl}
\frac{i}{R^2}\, K_{(p+1)0} & \to & e_+ & = & \frac{1}{2}\partial_+ \\
\frac{1}{R}\, K_{0i} & \to & e^*_i & = & \cos t^- \partial_{x^i} + \frac{1}{2}\sin t^- x_i \partial_+ \\
\frac{1}{R}\, K_{(p+1)i} & \to & e_i & = & \sin t^- \partial_{x^i} - \frac{1}{2}\cos t^- x_i \partial_+ \\
\frac{i}{R^2}\, J_{1q} & \to & e_+ & = & \frac{1}{2}\partial_+ \\
\frac{i}{R}\, J_{1i} & \to & f^*_i & = & \cos t^- \partial_{y^i} + \frac{1}{2}\sin t^- y_i \partial_+ \\
\frac{i}{R}\, J_{iq} & \to & f_i & = & \sin t^- \partial_{y^i} - \frac{1}{2}\cos t^- y_i \partial_+
\end{array} \tag{27}$$

where $x^i = r\Omega^i$ and $y^i = u\tilde{\Omega}^i$ parametrize flat Euclidean spaces. Together with the angular momenta K_{ij} and J_{ij}, they generate the isometries in the Cahen-Wallach space. They form a Heisenberg algebra

$$[\, e_i \,,\; e^*_j \,] = [\, f_i \,,\; f^*_j \,] = \delta_{ij}\, e_+ \tag{28}$$

with central charge e_+. There is one more Killing vector in this space,

$$\frac{i}{2}(K_{0(p+1)} - J_{1q}) \to e_- = \partial_- \tag{29}$$

The ground-state wavefunction

$$\Psi_0 = \Psi_0^{(\mathrm{AdS})} \times \Psi_0^{(S)} = e^{i(\Delta t - J\phi)} \cosh^{-\Delta} \rho \cos^J \theta \tag{30}$$

takes the form

$$\Psi_0 \sim e^{-i(p_+t^+ + p_-t^-)} e^{-\frac{1}{2}p_+(r^2+u^2)} \tag{31}$$

where $\Delta = \frac{1}{2}(-p_- + p_+R^2)$, $J = \frac{1}{2}(p_- + p_+R^2)$ (this notation differs slightly from [26]). The other states are created by repeatedly acting on Ψ_0 with the creation operators E_I^+, F_i^+, where

$$E_i^\pm \equiv e_i \pm ie_i^* = e^{it^-}(\partial_{x^i} \pm \tfrac{i}{2}x_i\partial_+) \quad , \quad F_i^\pm \equiv f_i \pm if_i^* = e^{it^-}(\partial_{y^i} \pm \tfrac{i}{2}y_i\partial_+) \tag{32}$$

The operator $E_i^\pm$ is the Penrose limit of the ladder operator $K_{\pm i}$ (similarly for $F_i^\pm$). Notice that this action changes the dependence of the wavefunction on t^- (shifting p_-) but leaves the quantum number p_+ unchanged. Thus these states span the eigenspace of $i\partial_+$ with eigenvalue p_+. This may be viewed as a constraint,

$$i\partial_+\Psi = p_+\Psi \tag{33}$$

and translations in the t^+ direction are then gauge transformations. Let us fix the gauge by choosing the gauge-fixing condition

$$\phi = 0 \tag{34}$$

which clearly intersects each gauge orbit once. Then the sphere S^q turns into the Euclidean space $\mathbb{E}^{q-1}$ (see eq. (25)) whereas this restriction does not affect AdS. The quadratic Casimir becomes

$$\begin{aligned} \mathcal{C} = \lim_{R\to\infty}\frac{1}{R^2}(\mathcal{C}_{\mathrm{AdS}} + \mathcal{C}_S) &= 2e_+e_- + (e_i)^2 + (e_i^*)^2 + (f_i)^2 + (f_i^*)^2 \\ &= \partial_+\partial_- + (\vec{x}^2 + \vec{y}^2)\partial_+^2 + \nabla_x^2 + \nabla_y^2 \end{aligned} \tag{35}$$

which is the Laplacian on Cahen-Wallach space.

To see how this is mapped onto the holographic screen (as $\rho \to \infty$), it is convenient to rotate the generators of the conformal group on the boundary of AdS (18),

$$\mathcal{K}_{AB} \to e^{-i\Delta t}\mathcal{K}_{AB}e^{i\Delta t} \tag{36}$$

The sole effect of this rotation is to shift the AdS Hamiltonian by its ground-state eigenvalue,

$$i\partial_t \to i\partial_t - \Delta \tag{37}$$

We obtain from (18),

$$
\begin{aligned}
\hat{\mathcal{K}}_{0(p+1)} &= \lim_{R\to\infty} \frac{1}{R^2}\, e^{-i\Delta t}\mathcal{K}_{0(p+1)} e^{i\Delta t} = \tfrac{1}{2} p_+ \\
\hat{\mathcal{K}}_{+i} &= \lim_{R\to\infty} \frac{1}{R^2}\, e^{-i\Delta t}\mathcal{K}_{+i} e^{i\Delta t} = -p_+ \Omega_i\, e^{it} \\
\hat{\mathcal{K}}_{-i} &= \lim_{R\to\infty} e^{-i\Delta t}\mathcal{K}_{-i} e^{i\Delta t} = e^{-it}\left(\nabla_i - i\Omega_i \partial_t\right)
\end{aligned}
\tag{38}
$$

where we used the parametrization $\Delta = \frac{1}{2}(-p_- + p_+ R^2)$. These vectors on the boundary of AdS form a Heisenberg algebra

$$
[\,\hat{\mathcal{K}}_{-i}\ ,\ \hat{\mathcal{K}}_{+j}\,] = \delta_{ij} p_+ \tag{39}
$$

with central charge p_+. They act on different spaces which are obtained as different limit of AdS space ($\rho \to 0$ for CW space; $\rho \to \infty$ for the holographic screen). AdS space provides a natural bridge between the two extremes but one would like to map one space onto the other without invoking the full machinery of AdS space. This map, if it exists, would establish the existence of a "holographic screen" on CW space.

To tackle this issue, first observe that the Hilbert space on CW space is much larger that the one on AdS space. It appears therefore that one ought to restrict the CW Hilbert space in order to compare with the holographic screen on AdS space. A convenient restriction is provided by the constraint

$$
\hat{\mathcal{D}}\Psi = -\Delta\Psi \quad , \quad \hat{\mathcal{D}} = r\frac{\partial}{\partial r} = x^i \frac{\partial}{\partial x^i} \tag{40}
$$

which selects the states of (fixed) scaling dimension Δ (eigenstates of the generator of scale transformations $\hat{\mathcal{D}}$). We must fix the gauge in the resulting theory. This is accomplished by restricting the CW space to a p-dimensional hypersurface. A convenient choice (gauge-fixing condition) is the cylinder

$$
r^2 = x^i x_i = \mathcal{R}^2 \tag{41}
$$

The parameter $\mathcal{R}$ is arbitrary. Notice that the flat-space limit is obtained as $\mathcal{R} \to \infty$. In this sense, $\mathcal{R}$ plays a role similar to the AdS parameter R. Poisson brackets turn into Dirac brackets. To compute them, we may solve the gauge-fixing condition (41), and parametrize the hypersurface,

$$
x^i = \mathcal{R}\Omega^i \tag{42}
$$

We may then use the parametrization of the coordinates to express the conjugate momenta on the hypersurface,

$$\mathcal{R}\frac{\partial}{\partial x^i} = -\Delta\Omega_i + \nabla_i \tag{43}$$

where we used the constraint (40) to write $\partial_r = -\Delta$. The CW Killing vectors (27) may be written as

$$\begin{aligned} e_+ &= \tfrac{1}{2}\partial_+ \\ E_i^\pm &= \frac{1}{\mathcal{R}}e^{\pm it^-}(-\Delta\Omega_i + \nabla_i \pm \tfrac{i}{2}\mathcal{R}^2\Omega_i\partial_+) \end{aligned} \tag{44}$$

where we used the definition (32) which is more convenient for our purposes. These vectors bare a striking resemblance to the generators of the conformal group on the holographic screen (18). To recover the scaling limit discussed above for the holographic screen ($R \to \infty$), which led to the modified operators $\hat{\mathcal{K}}_{AB}$ (eq. (38)), observe that in the limit $\mathcal{R} \to \infty$, the Cahen-Wallach metric (26) turns into

$$ds^2 = 4dt^+dt^- - \mathcal{R}^2(dt^-)^2 + \mathcal{R}^2 d\Omega_{p-1}^2 + ds^2(\mathbb{E}^{q-1}) + o(1/\mathcal{R}^2) \tag{45}$$

Diagonalizing,

$$4dt^+dt^- - \mathcal{R}^2(dt^-)^2 = -\mathcal{R}^2(dt^0)^2 + \mathcal{R}^2(dt^1)^2 \quad , \quad t^0 = t^- - 2t^+/\mathcal{R}^2 \quad , \quad t^1 = \frac{2}{\mathcal{R}^2}t^+ \tag{46}$$

we observe that if we impose the further restriction on the Hilbert space

$$\partial_1\Psi = \left(\frac{\mathcal{R}^2}{2}\partial_+ + \partial_-\right)\Psi = i\Delta\Psi \tag{47}$$

which replaces the contstraint (33) we imposed earlier, we may adopt one more gauge-fixing condition,

$$\frac{\mathcal{R}^2}{2}t^1 = t^+ = 0 \tag{48}$$

reducing the dimensionality of the hypersurface (41) by one. The resulting surface is a product of an Einstein Universe (conformally equivalent to the boundary of AdS) and a $(q-1)$-dimensional Euclidean space (equivalent to the flat-space limit of the sphere S^q after the gauge-fixing condition (34) is imposed). Taking the limit $\mathcal{R} \to \infty$, we obtain the operators

$$\mathcal{E}_+ = \lim_{\mathcal{R}\to\infty} e_+ = ip_+$$

$$
\begin{aligned}
\mathcal{E}_i^+ &= \lim_{\mathcal{R}\to\infty} \frac{1}{\mathcal{R}}\, E_i^+ = -e^{it^-} p_+ \Omega_i \\
\mathcal{E}_i^- &= \lim_{\mathcal{R}\to\infty} \mathcal{R}\, E_i^- = e^{-it^-} (\nabla_i - i\Omega_i \partial_-)
\end{aligned}
\tag{49}
$$

in agreement with the generators of the conformal group subalgebra on the boundary of AdS in the scaling limit $R \to \infty$ (38).

To summarize, I have recovered the algebra on the holographic screen in the scaling limit $R \to \infty$ (38) directly on the CW space, without making use of the AdS space. This was done by restricting the Hilbert space using the constraints (40) and (47). By fixing the gauge using the gauge-fixing conditions (41) and (48), a surface was selected in Cahen-Wallach space which in the scaling limit $\mathcal{R} \to \infty$ could be mapped onto the holographic screen (boundary of AdS) and the resulting algebra of the CW Killing vectors (49) agreed with the corresponding algebra (38) on the holographic screen. This provides a way of identifying a "holographic screen" on CW space, even though no holographic principle, similar to the one in AdS space, exists for CW space.

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